Prospectus: Second-Order Modal Logic

Andrew Parisi

Main Advisor: Marcus Rossberg

Committee Members: Lionel Shapiro, Donald Baxter, Jc Beall
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1 Introduction

This dissertation will explore the logical and philosophical consequences of defining the consequence relation in a specific proof-theoretic way. In particular, the goal is to lay down constraints for a set of rules to determine the meaning of a logical constant. Within these constraints I will explore the system, or family of systems, that blend second-order logic and modal logic. Both of these systems have been explored by philosophers and logicians, but a smaller number has explored the combination of the two systems. Ruth Barcan Marcus [3] and following her Timothy Williamson [101, 104] have done some work in this area, but it ought to be extended. Furthermore, the work that Barcan Marcus has done was using an axiom system, and the work that Williamson has done is model theoretic. There has been no approach to the questions in terms of a natural deduction system. This dissertation aims to begin the exploration of this potentially useful resource. Modal logic has a long philosophical history and, despite the protests of Quine [76, 77, 71, 73], has become commonly accepted as logic. Second-order logic, on the other hand, has had a much more difficult time overcoming Quine’s criticisms [75]. Recently though, several authors are beginning to look more closely at Quine’s arguments, and developing ways of showing that second-order logic is proper logic [11, 89, 93, 87, 107]. My dissertation will address these issues. I will not argue for a view of what logic is, but develop a view of second-order modal logic given one conception of logic. This will mean developing a reasonable account of how logical constants get their meaning, which will mean providing a uniform account of first- and second-order quantifiers (if one is necessary), and providing uniform accounts of modality across the various systems of
modal logic (if one is available). The goal is to give a formal analysis, given certain views on logic and language, of the shape a logic must take. Since the project is concerned with modal logic and quantification, the I expect the results of this exploration to give answers to questions as to what our ontological commitments are, what possibilia are, and the nature of modal properties.

1.1 Quantification

I will begin the dissertation by exploring issues concerning second-order logic. My main concern will be with the potential ontological commitment that is incurred by second-order quantifiers. However, I will explore other arguments that have been made against second-order quantification. I will then explore the stances that can be taken regarding these arguments. Two of which I will reject out of hand because they accept that a weighty ontological commitment is incurred by second-order quantification. This will leave my view and another view that I will argue against in the following chapter of the dissertation.

The view that I will argue against is that originally developed by Boolos [11], but advanced more recently by Lewis and others [54, 41]. This is the view that second-order quantification, as it is normally understood, is ontologically innocent. This ontological innocence is established via a translation of monadic second-order logic into the language of plurals. This section will explore issues of the preservation of ontological commitment across translation [94, 1]. Furthermore, I will explore whether or not the translation is successful, and if it is, do plural quantifiers really not carry ontological commitment to plurals. I will argue, that if they do not, then
they cannot be as strong as second-order logic. If they do, then there is no advantage to be had.

After arguing against the plural interpretation of second-order logic, I will present my view. My view has developed out of the view that Sellars advanced in *Grammar and Existence: A Preface to Ontology* and *Naturalism and Ontology* [89, 91]. This discussion will bring explore Sellars’s response to Quinean arguments, and the view that results from these responses. I will argue that quantifiers so interpreted need not carry ontological weight, but that first-order quantifiers do in virtue of other considerations.

### 1.2 Logic

I will then consider logical constants more generally. Because it is generally admitted that first-order quantifiers meet the constraints for an expression to be properly logical, I will take for granted that they are logical. The proof-rules for second-order logic being analogous in every important way to first-order quantifiers, they will be admitted as logical vocabulary. Although there will need to be some exploration of Comprehension Axioms here. From this discussion I will give a general account of the necessary conditions for an expression to be logical. I will not dwell much on this issue, but only use it as a my unargued-for starting point. As with any investigation, there have to be things that are taken for granted in order to move forward. This will be something that space will not permit much focus on.
1.2.1 Necessitism and Contingentism

Quantification and modality has been the nexus of the debate of contingentism and necessitism (to use Williamson’s words). I will see how the logic I have developed weighs in on this debate. Necessitism is the claim that, “necessarily everything is necessarily something” [103]. Contingentism is “the negation of necessitism” [103]. As I understand these two statements, necessitism amounts to the view that everything that exists exists in the actual world. Whereas contingentists deny this, there are being at other worlds that are not actual. Williamson considers this debate to be the successor to the debate over actualism and possibilism. The debate between the view that all entities were actual, and its rival view that there are merely possible entities. Williamson also argues that a necessitist interpretation of modal logic is difficult to avoid in the second-order case [104]. This is because the combination of second-order logic and modal logic yields that $\exists X \Box \varphi \leftrightarrow \Box \exists X \varphi$.

1.2.2 Gödel’s Ontological Proof

Modal second-order logic has also been put to other uses, Gödel famously made use of it to formulate an ontological argument [36]. Roy Cook and Alan Hazen have written on this topic [20, 38]. This dissertation will explore any connections between different debates that second-order modal logic has been used to advance.

This dissertation will carefully consider the debates about second-order logic from the past. I will explore the views on second-order quantification, and quantification in general of Quine [75], Sellars [91], and Dummett [24]. I will also explore past views on modality, and how it interacts with quantifiers. Here I will make use of
work by Sellars [91], Carnap [17], and Barcan Marcus [3]. Overall the strategy that I adopt here is to explore what logic is possible given what language is like. I adopt a view of language that I will not argue for, but that will constrain what can count as logical, and what shape logic must take if language is the way that I take it to be. This methodology is familiar in philosophy, and I see myself as continuing the tradition of exploring the apparatus of our contact with the world to see what the world must be like. The project goes along the lines that Dummett suggests in The Logical Basis of Metaphysics [26]

It [metaphysical controversy] is therefore to be resolved if not within logic properly so called, then within that part of philosophy of which logic is a specialized branch: the philosophy of thought, which when approached via language, becomes a theory of meaning [pg. 15].

2 Second-Order Logic: History of the Debate

2.1 Objections to Second-Order Logic

There are several arguments that are given for the claim that second-order logic is not logic. These, however, can be roughly organized into two groups: (1) The ontological commitments that are incurred by second-order quantifiers is unacceptable. (2) Second-order logic is mathematics, and mathematics is not logic. There are some arguments that blur the lines between (1) and (2), but for the most part they fall between the extremes of (1) and (2).

Both arguments point out the requirement that logic be topic neutral. In the first case, second-order logic comes with ontological commitment, and so begs the
question against nominalism. A weaker, though potent form of this objection is to argue from the truth of nominalism to the rejection of second-order logic. For this argument to be sound, nominalism must be true. However, barring an argument that it is false, it is open for nominalists\(^1\) to reject second-order logic. There are ways of reading Quine as objecting to second-order logic in one of these ways [91], but there are others who can be seen as arguing that second-order quantification is inconsistent with some brands of nominalism [24, 33] As I noted above, these two objections are connected. Quine traces the argument by asking what the values of the second-order variables could be. Upon discovering that the only viable option is sets, he concludes that second-order logic must be a disguised version of set theory. There are two important and distinct points that need exploration though.

The second argument points out that second-order logic is too closely tied to the practice of mathematics, in particular to set theory. The motto of this argument is the claim that second-order logic is “set theory in sheep’s clothing” [75]. One way of understanding the argument is that second-order quantifiers have as values sets, and so it really is just set-theory in disguise. Another way is that if we call second-order logic what set theory we choose will change what is logically true: If we take our second-order consequence relation to be set by the standard semantics, where the range of the \(n\)-ary second-order variables is the powerset of the \(n^{th}\) Cartesian product of the domain, the consequence relation will change depending on what set theory is selected for the model-theory. This is because of the capability of the second-order quantifier to express the background set theory [57]. If the consequence

\(^1\)Here I am using the term ‘nominalism’ to denote the medieval view about properties, not the modern view about abstract objects generally.
relation changes based on the set theory that is selected, it seems like mathematical questions, for example about the truth of the generalized continuum hypothesis, are bound up in second-order logic. If the former is true, it seems like logic cannot be topic neutral, since the mathematical theory one subscribed to will dictate one’s logic. My approach to second-order quantification will also quell this objection.

Both these objections are based on the thought that what takes priority in determining the semantics of a logical constant is the way in which that constant comes to be true. I will not dispute that this is an important part of the semantics for a constant, but it must come only after other considerations have been made, considerations about what inferences are legitimate for logical constants. This being the case, the first point is that the above argument is relying on a view by which quantifiers must have ranges. If this can be rejected, then there is no reason to require sets to be their variable. At this point, it may be possible to either propose another entity to be in the range of the second-order variable, or to show that not all quantifiers require a domain over which to range.\textsuperscript{2}

\section*{2.2 Views on the Commitments of Second-Order Logic}

The debate over second-order logic thus has roughly four camps: (1) Proof-theoretic/syntactic approaches legitimizing second-order logic, such as those advocated by Barcan Mar-

\textsuperscript{2}Here is an argument to be worked out in more detail: the manifestation requirement on a logic entails that a proposed model-theoretic semantics for a logic have a complete proof system. The question is whether that claim relies on language being recursive. Or does manifestability itself have recursiveness built into it? The important point is that the logical constants are determined via proof rules, and not from rules along with axioms. There are only axioms that can be shown to be true in virtue of the theory of meaning that is adopted. At the moment, the only axiom I am admitting is $AX—\Gamma, \varphi \Rightarrow \varphi, \Sigma$. This is because I hold that it is irrational to both assert and deny $\varphi$ \textsuperscript{[82]}.}
cus, Sellars, et al. [4, 89, 91, 87, 83]. (2) Those that try to preserve a model-theoretic account of second-order logic but show that it is ontologically innocent. Proponents of this view include Boolos, Lewis, et al. [11, 12, 54, 41]. (3) Those who claim it is illegitimate such as Quine, Koellner, et al. [75, 47, 44]. (4) Finally, there is the view of Shapiro [93, 96], who admits that the criticisms of (3) are correct but thinks this should not stop us from making use of second-order logic.

The previous section is an argument that position (3) is not well supported. Position (3) rests on the claim that second-order logic either has unacceptable ontological commitments or that it is not topic neutral. In the previous section I will have argued that this is not the case, and so there is no reason to prefer position (3) if other positions can be made to work, for instance position (1), which is where this dissertation falls. Position (4) requires position (3) to be true, and so the previous section will also provide reason to reject position (4). The rest of this chapter will be an exploration of position (2), and ultimately an argument that it fails. This being the case, position (1) is the best option available with respect to second-order logic.

I should be careful here. This argument is not deductive. It is possible to think that there is no argument to deny that second-order logic is not logic, but that it nonetheless is not. Here I will invoke Carnap’s Principle of Tolerance [17, §12], or something like it. This argument follows something along the lines of Sellars [91]. Though I think that it is necessary to argue along these lines given that settling the question of what are the necessary and sufficient conditions on an expression being logical are difficult, if not irresolvable. Since second-order logic meets all the necessary requirements of being logic, there is no reason to deny it that status. This
being the case, showing that (3) is false is enough reason to move to (1) barring an argument that second-order logic does not meet the necessary requirements for an expression to be logical.

So this chapter will be concerned with outlining the details of the debate, and showing that the arguments against second-order logic are flawed. This rules out positions (3) and (4) above. The next chapter will argue against position (2).

3 Boolos: Monadic Second-Order Logic and Plural Quantification

As I mentioned above, there are roughly two ways to avoid Quine’s accusations that second-order logic is not logic. The first is to attempt to show that quantification is not the touchstone of reference, or that quantification need not be ontologically committing. This is the view that I take, and the one that will be filled out in the following chapter. Alternatively, one could accept the philosophy of logic that Quine relies on, but show that second-order logic does not commit one to more than the first-order quantifiers. The idea here is that when we move to second-order logic we introduce new quantifiers that range over the first-order entities in a different way than the first-order quantifiers do. This is accomplished by translating second-order logic into languages which are normally thought not to involve ontological commitment. In his 1984 article, To Be is to be the Value of A Variable (or to be Some Values of Some Variables), Boolos argues that second-order quantifiers do not commit one to more than one is already committed to by the first-order quantifiers by giving
such a translation from second-order logic into a system of plural quantification. I take it that he here has the idea, along the lines of Shapiro [94], that a translation from one logic to another preserves ontological commitment. As mentioned above, plural quantification is supposed not to commit one to more than a first-order theory would. From this, Boolos concludes that second-order logic must have the same commitments.

This chapter will provide a more in depth analysis of this argument, and discuss contemporary approaches to interpreting second-order quantification with plural quantification. I would like to present the argument in the form of premises and a conclusion. That would go *something* like the following, although this would require some tinkering:

1. Plural Quantifiers do not require ontological commitment to entities not required by first-order quantification.

2. If there is a translation from one logic into another that preserves the consequence relation, then the two logics have the same ontological commitments.

3. There is a translation from Plural Quantification into Monadic Second-Order Logic\(^3\)

C. Therefore, second-order quantification does not require ontological commitment to entities not required by first-order quantification.

\(^3\)I should note that Simon Hewitt has written a dissertation on accommodating full second-order logic with plurals/superplurals [42]
This argument seems to be valid, so one of the premises must be false. I will argue that all three premises are open to doubt. The first premise has been considered by Ben-Yami [7], and Resnik [81]. Premise (2) is one that requires scrutiny. In general reducing one class of statements to another shows that there need be no ontological commitment required by the first class that is not already required by the second. For example, if the statements of chemistry could be reduced to those of physics, there would be no reason to think that there was more to chemical truths than the physical truths. Any commitment of the chemical theory would be explainable in terms of a commitment of the physical theory. However, it is difficult to see how this works in the case of logics. It is widely accepted that the commitments of intuitionistic logic are different from those of classical logic [24], but there is a translation from one into the other. So this premise requires more attention. Boolos [11] has given good reason to think that (3) is true. However, there are arguments that when the translation is extended to modal logic, it fails [55].

3.1 Plural Quantification and Ontological Commitment

With respect to the first-premise we can consider the comprehension axiom for plural quantification: $\exists x \varphi(x) \rightarrow \exists x x \forall y(y < x x \leftrightarrow \varphi(y))$. This is informally read as, “if there is a $\varphi$, then there are some $x$’s (being represented by the plural variable $xx$) such that for any $y$, it is one of them (the $x$’s) if and only if it is a $\varphi$.” It is hard not to think of the $x$’s as a group, or some other reified entity. Baxter [6] has suggested ways of understanding the plural quantifier so that it does not commit one to more than the ordinary first-order quantifier. This needs to be carefully explored.
Furthermore sentences such as ‘All the children formed a circle and sang’ [7, pg. 215] pose a problem for thinking that there are separate quantifiers for pluralities. The above sentence is supposed to in one instance treat ‘all the children’ as a plural subject of ‘formed a circle’, but ‘sang’ seems only to take singular subjects since groups only sing in virtue of all their members singing. It must be argued that plural quantification comes without ontological commitments, if it can even be made to be coherent. This section will explore whether plural quantification makes sense, and if it does the extent to which it is plausible that it carries no more ontological commitment than ordinary first-order quantifiers.

3.2 Translation and Explanation

At this point it will be important to explore the relationship between translation and ontological commitment. Alston [1] and Shapiro [94] have both considered this approach to ontological commitment. The Boolos translation is supposed to work as follows, we have a language whose ontological commitments we are unsure of, but we can translate it into a language whose ontological commitments we are sure of. From this we can learn something about the first language. There are several places where this line of reasoning needs to be fleshed out. It is unclear why we should think that a translation shows that the first language does not have commitments. It needs to be shown that such a translation is not revealing that we are unsure of the ontological commitments of the second language as opposed to suring up the ontological commitments of the first. Here I hope to argue that if a translation is going to have direction, as in (2) above, the second language must somehow be seen
as explaining the first. Consider again the case where a translation is given between
the language of chemistry and the language of physics. The most that this tells us
is that each language can express the other while preserving whatever notion we
would like to preserve, in this case it would most likely be truth. That is, for every
sentence of chemistry, \( \varphi \), there is a sentence of physics, \( \psi \), such that \( \varphi \) is true iff
\( \psi \) is true. This does not tell us that we can get by only with physical truths by
itself. This could show that the language of chemistry has commitments that we
were unaware of. Even worse, if there is a translation goes in the other direction,
as there is in Boolos’s case,\(^4\) there is no reason to think that the second-language is
not being reduced to the first. So in Boolos’s case, the translation may reveal that
plural quantification requires the existence of sets. This is addressed is why it is
that two languages one of which can be translated into another cannot have different
ontological commitments. This question is explored by Wright [106]. Michael Resnik
has also argued against this view [81].

I will argue that the reason we think that a such a translation does tell us something
about the ontological commitments of the first language is because we take the
translation to be explanatory. In the case of reducing the chemical language to the
physical language, we would understand the reduction as giving us an explanation
of the subject matter of the chemical sentences. We could read chemical sentences
as abbreviations of complicated processes in the physical description of things. So
in order for the argument to go through the translation the Boolos provides must

\(^4\)I think this is true, but it is worth checking. It would be hard to see how there could be a sentence
of plural quantification that is not a sentence of monadic second-order logic, since to go back
all we would have to do is consider \( \prec \) to be our ‘predication’ relation.
also explain what second-order quantifiers are. It is contentious how we should understandBoolos’s distinction. This amounts to requiring that the explanation be given what Hewitt calls ‘maximal understanding’. This he opposes to ‘minimal’ understanding, in which the translation serves only to show us what the expressive power of second-order logic is. He concedes that, “whilst the minimal understanding does not empty the Boolosian interpretation of all philosophical significance, it does deprive it of importance for debates about the nature and logicality of second-order systems and the semantics of predication”[41, pg. iv]. The above argument lends credence to this statement.

3.2.1 The Cardinality Problem

Here I will argue that there are problems with understanding plural quantification as explaining second-order logic. The first of which will depend on how the structure of second-order logic depends on the underlying set-theory. A crucial fact about models of second-order logic is that it can characterize up to isomorphism structures of different sizes of infinity. This, in part, requires an interpretation whereby there are always more objects in the range of the second-order quantifier than there are in the first. The problem here is how plural quantification can account for this fact. If plurals do not commit us to entities over an above the entities that we were committed to in a first-order language, how can there be strictly more plurals than there are first-order entities? The argument is going to go something like this: Second-order consequence requires there to be more objects that are in the range of the second-order variables than there are in the first order variables. If this is the
case, then the sort of thing that are in the range of the first-order variables cannot be all the things that are in the range of the second-order variables, since there are more of the latter group. So the translation from first-order logic into plural quantification cannot be an explanatory one, since it fails to explain the nature of the objects that are in the range of the second-order quantifiers.

3.2.2 Predication and ‘is one of’

Another worry about the legitimacy of the translation comes from the fact that it distorts what is normally thought of as predication. If we are taking plural quantification to help explain issues like predication, then there is a regress problem. Consider the sentence, $Fa$, and suppose it is true. On the Boolos interpretation, this is at least in part explained by the fact that $a \prec ff$ is true. For this translation to succeed, we need to be able to go between ‘$F$’ and ‘$ff$’. For convenience, given a predicate ‘$F$’, I will write the corresponding plurality as $\iota Fx$. $a \prec ff$, will then be written as $a \prec \iota Fx$, which can be more perspicuously written as $P(a, \iota Fx)$. This, however, will in part be explained in terms of the ‘is one of’ relation holding between ‘$a$’ and ‘being one of the ‘$ff$’: $a \prec \iota P(x, \iota Fx)$. Again, this can be more perspicuously written as $P(a, \prec \iota P(x, \iota Fx))$. If this were a simple case of abstraction this would not be a problem. However, each iteration of the ‘is one of’ relation is meant in part to explain the previous one. So our explanations will never come to an end. Any explanation we give of a predication in terms of the $\prec$ relation, will also require an explanation, one is forthcoming, but it too will require an explanation. So a full explanation of the truth of a sentence will never be found.
3.3 The Translation Itself

Finally, I will consider whether or not the translation itself succeeds. While it is true that Boolos’s translation works for the fragments of the languages considered. It is unclear that it will work when the languages are embedded in other contexts. In particular, if modal logic is logic then we cannot treat second-order quantifiers in isolation from these other logical connectives. Here I will begin to venture into second-order modal logic. Linnebo [55] and Williamson [102] argue that Boolos’s translation fails because it does not account for the behavior of plurals in modal contexts. He argues as follows: If plural referring expressions are like names, then we have good reason to accept the following inference as valid:

\[
\frac{a \prec bb}{\Box a \prec bb}
\]

i.e. from \(a\) is one of the \(b\)’s we ought to be able to infer that \(a\) is necessarily one of the \(b\)’s. The idea is that names of pluralities refer rigidly, as do names.\(^5\) If this is an inference that is licensed by plural-quantification, then we should expect the corresponding inference in second-order logic to be valid:

\[
\frac{Fa}{\Box Fa}
\]

\(^5\) I will argue later in the dissertation that rules governing identity can validate the inference, and that such rules are acceptable given the necessary conditions that I will propose for a logical constant. In particular, they will be rules about substitution of names in certain contexts. That being the case, it will require further arguments than those that I will present to adjudicate between the validity of the inference from \(a = b\) to \(\Box a = b\). I should note: this does not entail that I am a logical pluralist, though that view is still open to me. As I wrote above, I will only set down necessary conditions on what can be a logical expression, what I set down may not be exhaustive, and neither need they be sufficient conditions. One of the goals is to see how much philosophical mileage can be had by setting down some plausible, though perhaps debatable, necessary conditions on what a logic must be like.
As I will show in the dissertation, doing this will require a very strange modal logic, and make it impossible to capture some natural systems of modal logic like S4 and S5, since in the presence of the reflexivity axiom, $\Box \varphi \rightarrow \varphi$, this becomes Triv [43], a system that is equivalent to second-order logic without modality. This is good reason to reject that inference as a requirement on a logic, which is problematic if the corresponding inference is a requirement on the logic of plural quantification.

Having not yet decided if this argument is successful, I will explore both sides of the debate in this section. I will explore the issue to see what can be said. The relevant literature on this debate comes from Williamson [102, 103]. Simon Hewitt [41] in a defense of the plural quantification interpretation of modal logic argues that we should see the issue as reducing to a debate over inferences involving disjunction because identity statements about plurals can be thought of as disjunctions of statements of identity about their members.

4 Quantification, Topic Neutrality, and Ontological Commitment

Here I will discuss my own view of the relationship between quantification and ontological commitment in general. This section will spell out what I take a quantifier to be, and how I take second-order quantifiers to come with ontological commitment, but first-order quantifiers not to. This will begin with an exploration of Sellars’s views on the topic.
4.1 Sellars and Quantification

I will begin by discussing Sellars’s view as expressed in Grammar and Existence: A Preface to Ontology, and Naturalism and Ontology. Here I will present the ways that Sellars avoids the arguments that were presented in the earlier chapter, though it is unclear how he would respond to the charges of mathematics. I will look at the historical debate closely for one chapter. In particular, Sellars took himself to be responding to arguments both from Quine [75], and both Geach [33] and Dummett [24]. I will argue that Sellars successfully sustains objections against these arguments, and that so doing allows the adoption of position (1) above. This, however, will raise questions as to how we are supposed to be reading quantifiers, and what the existential quantifier is if it does not of necessity require an ontological commitment.

This section should focus on Grammar and Existence, since that is a gold mine of philosophical ideas involving the ontological commitment involved with singular quantification, but not quantification generally. The argument is very roughly this: we make a mistake in reading $\exists x W x$, as ‘there is an $x$, such that $x$ is $W$’. In particular, this is not a well formed sentence in English: $x$ is being treated both as a predicate word, and as a singular term. In fact, I have trouble finding a way to read $\exists x W x$ with ‘there is’ unless the translation is simply (1)‘there are W’s’, or (2)‘there is a $W$’, or perhaps, (3)‘there is a thing, $x$, such that $x$ is $W$’. The better reading is ‘something is $W$’. It is this reading that I think, as does Sellars, should be primary.

Notice in the corresponding case of $\exists f f a$, we cannot read it in any coherent way using the locution ‘there is’. If we want to be consistent with the above renderings of the statement, we would have to choose one of the three: (1a) ‘There are a’s’, 
(2a) ‘There is an $a'$, or (3a) ‘There is a thing, $f$, such that $a$ is $f$’. The only one of these that is plausible is (3a), but even that cannot be right, since things are not things, things are ways. It is incoherent, unless the above ‘is’ is the ‘is’ of identity to say that ‘$a$ is $f$’. But of course that is not the ‘is’ of identity, it is copular.

A plausible reading of $\exists \, f \, f\! a$, along the lines of the proper reading of $\exists x W x$ is ‘$a$ is something’, or perhaps the even more natural ‘$a$ does something’. This reading, I will argue does not commit us to things (properties, qualities) that other things are, or things (properties, qualities) that other things have.

4.2 Quantification as Generalization

The first point to notice is that because I require the introduction and elimination rules for the universal and existential quantifier to determine their meaning and I require these rules to be recursively specifiable, I am committed to a consequence relation for second-order logic that has as a model-theory the Henkin models, as opposed to the standard model theory for second-order quantification. Immediately, I will note that although these models are isomorphic to models for a two-sorted first-order logic [57], this is not the proper characterization of what is going on. A two-sorted first-order model has two sorts of things between which there is, perhaps, a ‘predication tie’ for atomic sentences. At least for the moment, predication is something that is primitive, and can be understood in terms of a ‘predication tie’, or in the Henkin models as ‘being a member of’, but this is only an equivalent formulation of what is happening for atomic sentences, it is not an explication of it.

---

6 This is a departure from Sellars’s choice of ‘$a$ is somehow’ in [91]
There are several more pressing philosophical points that need to be addressed as well. The first is that if this argument is going to work it has to be the case independently of the second-order sentence that the primary reading of $\exists x W x$ is ‘something is $W$’. I am not sure that Sellars adequately establishes this, and so I will have to do some research as to how ‘something’ and ‘there is/there are’ are related. Hopefully, once the theory has been developed, I will be able to show why the inference from ‘something is $W$’ to ‘There are $W$’s’ and back again is permissible. But that the inference from ‘$a$ does something’ to ‘there is a thing that $a$ does’ is not. This aspect of the argument at this point looks very difficult. So I will either have to find such an argument, or I will have

The second place in this argument that needs fleshing out is what a quantifier is, if it is not a device for quantifying, i.e. for picking out objects. Here I will argue that the proper reading of $\exists$ is something, in order to get the bare ‘some’ we require two kind terms, e.g. some $F$ is $G$. It may be that, historically, the locution ‘some $F$ is $G$’ came first, which is why in order to have a bare generalization with one quantifier it made grammatical sense to say ‘some thing is $F$’. Here ‘thing’ is a kind term, but a special kind. It is a Carnapian dependent universal word.[17] So claiming that ‘some thing is $F$’, $\exists x (Tx \land Fx)$, amounts to claiming $\exists Fx$, since everything is a thing, $\forall x Fx$. (Analogous things can be said about the universal quantifier, $\forall$.). However, once we introduce second-order quantifiers into our language, ‘thing’ is no longer a dependent universal word, but an independent universal word, and so we cannot in general go from ‘something is $F$’ to ‘some thing is $F$’. [89]  

\[7\]I should note that much of this discussion is prefaced by Sellars in Grammar and Existence: A Preface to Ontology, but this section of the dissertation serves to clear up certain aspects that
Finally, I will have to account for the fact that $\exists x Fx$, carries ontological weight, but $\exists f f x$, does not. Here I am going to understand sentences of the form ‘$a$ exists’, and ‘$W$’s exist’. These sentences I will take to be metalinguistic. I want them to entail the sentences $\exists f f a$, and $\exists x W x$, respectively, the first of which will commit us to the existence of individuals, the second to the existence of individuals of a certain kind. Neither of which will involve commitment to properties, qualities, etc.

4.3 Quantification and Mathematics

There are points that were addressed in §2.1 that Sellars did not address, and that I have not shown that my view of second-order quantifiers can get around. SS4.1-4.2 are meant to show that second-order quantifiers need not commit us to the existence of properties, attributes, etc. The issue of the relation between mathematics and second-order logic still needs to be addressed. In particular, there are two objections to second-order quantification that I noted above, though this may grow as my research does:

1. Second-order logic is not topic neutral since it is really just set theory.
2. Second-order logic is not logic since the consequence relation will change depending on which set theory is adopted for the model theory.

With respect to (1), as I mentioned above, the quantifiers are divorced from having their meaning determined by their ranges. Their meaning is determined by their introduction and elimination rules. Here it will also help to mention objections to this claim that have been given by Boolos [9], that there are validities of second-order logic that are not validities in set theory, and vice-versa that my account of second-order quantification does not even prove that there is an empty set. Further problems arise in considering polyadic second-order logic [87]. The claim that logic is just set theory thus seems problematic.

With respect to (2), second-order logical consequence is determined by the introduction and elimination rules for the logical connectives. This is independent of the underlying set theory in which the models for second-order logic are given. So varying our underlying set theory will not change the logical consequence. Defining logic in this way separates it from mathematics in a way that makes it unvarying between different conceptions of the a mathematical universe.

5 Logic

When discussing the introduction and elimination rules for a second-order quantifier, there is little reason to doubt that they satisfy the conditions for logicality if the first-order ones do. This being the case there was little reason to discuss what conditions for the introduction and elimination rules of a logical connective I consider necessary
to determine the meaning of that connective. Moving forward, however, this will be necessary because the proof theory of modal connectives is much less obviously logical. It is generally an important question to ask whether or not the given rules for a modal expression can be said to determine the meaning of the expression.

I note here that I hold that the introduction and elimination rules at most determine, or give the outline of the meaning of the logical expression, I do not think that they constitute the meaning of the logical expression. They determine the meaning in the sense that they provide what is required for the recognition of that connective as (1) logical, (2) distinct from other logical connectives, and (3) communicable.\(^\text{10}\)

I will not argue for the three constraints above, I will also not argue that they are the only plausible necessary constraints on proof rules determining the meaning of a connective, not will I argue that they are sufficient. The goal is to see what metaphysics comes from a philosophy of language that takes on these constraints. Arguing that this is a plausible philosophy of language is a task for another paper. This section will have to first discuss these three constraints, and explore comprehension schemas to see which, if any can be counted as logical. This chapter will therefore have to deal, or at least briefly discuss the notion of impredicativity.

\(^\text{10}\)This last requirement as stated is vague, I will make it clearer as I move forward. The general idea is that there must be conditions on the connective that make it possible for that connective to be isolated in a context, so that it cannot be confused with itself or other symbols. This too is vague, but the idea is to rule out rules such as double negation elimination as meaning determining. In that rule there are two negations, and so there is no reason that it should be seen as an elimination rule for one connective \(\neg\) for which no introduction rule has been provided, or whether it is the elimination rule for one connective \(\neg\), which cannot eliminated in isolation. Both cases are meant to be problematic.
5.1 Logical

The first necessary condition on an expression’s meaning being determined by the introduction rules is that the connective be thereby recognizable as logical. Let $\odot$ be a purportedly logical expression, and $\varphi(\odot)$ be an expression containing $\odot$ as the primary part of that expression, e.g. the main connective/ operator in a sentence. The connective is recognizable as logical when the contexts that involve the assertion of $\varphi(\odot)$ and contexts involving the denial of $\varphi(\odot)$ can be reduced to contexts that make no explicit mention of $\odot$. This might be expressed as saying that the truth and falsity of the $\varphi(\odot)$ is determined by the truth and falsity of the constituents of $\varphi(\odot)$. This amounts to two conditions on the introduction rules for the sequent calculus.

Here I will argue for the structural rules that must be reduced to atomic cases. At the moment, I suspect that this will be all of them, though without a deeper explanation of what ought to count as logical, I am not sure yet. I also suspect that cut eliminability will be required for a connective to be meaning determining. Each of these is meant to be based on the thought that there is nothing more to the logical connectives than what can be said about the assertion and denial of their parts. Furthermore, the cut elimination theorem for a logic serves as a consistency proof, and so can be seen as a way of ruling out the infamous Tonk rules [68] as determining the meaning of the connective. These constraints are taken from discussions of logicality taken from Hacking, Prawitz, and Dummett [37, 67, 65, 26]

It is hoped that I will be able to explain the concept of topic neutrality in terms of these notions. The idea is that expressions are topic neutral when their assertion and denial conditions can be reduced to assertion and denial conditions of sentences
specific to that domain.\textsuperscript{11}

5.2 Distinct from Other Connectives

One of the notions that leads to constraints (2) and (3) is that the meaning of a logical connective must be exhibitable, or manifestable. This idea, following Dummett [26, 27], is that there can be no part of the meaning of a logical notion (or any other linguistic notion for that matter) that is not displayable in terms of some use we make of it. So logical connectives must be isolable from one another based on the inferences that determine their meanings. This rules out the following inferences as determining the meaning of the connectives involved:

\[
DN \quad \frac{\Gamma \Rightarrow \varphi, \Sigma}{\Gamma \Rightarrow \neg \neg \varphi, \Sigma} \quad \text{DeM} \quad \frac{\Gamma \Rightarrow \neg \varphi \land \neg \psi, \Sigma}{\Gamma \Rightarrow \neg (\varphi \lor \psi), \Sigma}
\]

\(DN\) cannot be meaning determining for the above mentioned reasons. It fails to show which connective’s meaning is actually being determined. If this were assumed to be a good introduction rule for determining the meaning of a connective, it would determine the meaning of the connective \(\neg \neg\). The second sentence fails to feature only one connective that is being introduced. Both \(\neg\) and \(\lor\) are introduced in \(DeM\). This is problematic because if the rules governing the use of a connective can be manifested, then it should be possible to isolate that connective and ask about the rules determining \textit{its} meaning. Were this rule a meaning determining one, it

\textsuperscript{11}Talk here of assertion and denial conditions is only meant at the moment to be a term of convenience. I do not want at this point to be leaning the discussion towards different types of use based theories of meaning. I am hoping that ‘assertion and denial’ conditions can be replaced with whichever concept is preferred by a use-based theory of meaning, e.g. proof conditions, denial conditions, consequences, etc.
would determine the meaning of \( \neg (\lor) \). It is problematic to not be able to ask what determines the meaning of a connective in isolation from other connectives.

### 5.3 Communicable

The communicability requirement is also related to the manifestability requirement had by Dummett. Though this requirement is more general. This means that it must be possible to communicate the rules that determine the meaning of a connective. So in particular, the set of inference rules governing the connective must be recursively enumerable. Otherwise, there would be no way for a finite being to communicate the rules determining the meaning of the connective. Because of this constraint, the standard semantics for second-order logic are ruled out as defining inference rules that are meaning determining. That is, if a model-theory is given, it will only determine a logic if that logic has a complete proof theory, wherein those proof rules can be said to determine the meaning of the connectives.

### 5.4 Comprehension Schemas

Here I should look at ways of formulating the Comprehension Schema. This will likely involve the use of the lambda calculus. I suspect that the rules defining the lambda operator will be as follows:

\[
\begin{align*}
L\lambda & \quad \frac{\Gamma, \varphi(a) \Rightarrow \Sigma}{\Gamma, \lambda x \varphi(x)[a] \Rightarrow \Sigma} \\
R\lambda & \quad \frac{\Gamma \Rightarrow \varphi(a), \Sigma}{\Gamma \Rightarrow \lambda x \varphi(x)[a]}
\end{align*}
\]

The \( \lambda \) operator turns a sentence with one free variable into a predicate. The
quantification rules will be as defined in the first-order case. The comprehension schema will be defined by what predicates can be formed from sentences. At this point I have no reason to suspect that the above rules are not logical, unless they do not admit of cut-elimination or the reduction of other structural rules, though I suspect that they do. A potential worry, however, is that impredicative uses of the \( \lambda \) operator will not allow the rule Weak to be reduced only to atomics.

Here again, results from Hacking [37], will be very useful. He proved that ramified type theory could be incorporated on this view of logic. I suspect that I will diverge from his views at some points, and so will have to re-investigate some of these issues, but his work provides an excellent place to start.

6 Modal Proof Theory

6.1 Motivation

The trend this century and much of the last was to approach modal logic model-theoretically. This had beginnings in Carnap [16], but achieved its height with Kripke [48].\(^{12}\) There has been much less work on syntactic approaches to modal logic, especially with respect to sequent systems, the exceptions being [63, 100, 2, 65]. Following the methodology that introduced in the previous chapter, I will use the systems developed here to explore modal properties.

At this point I will address historical explications of modality using a syntactic approach. C. I. Lewis was one of the first philosophers to approach modality in

\(^{12}\)For an account of the development of possible world semantics see[21].
a logical setting [51]. His approach in keeping with the time that he worked in was syntactic. Lewis developed several systems that are weaker than system K. An exploration of these logics in the sequent calculus may be fruitful, though at this point I am uncertain. My approach here is opposed to the model-theoretic approach that was mentioned above.

I will use my approach to logic to explore the connection between modal logic and quantified logic. For example, at the moment, the most natural formulation of rules for system K in a sequent calculus, when combined with the natural rules for quantification result in a system that is a hybrid between contingentist and necessitist. As I said above, the necessitist claims roughly that any object that exists necessarily exists, the contingentist denies this. What many have taken to characterize the necessitist view is their adherence to the Barcan formula, (□∀xφ → ∀x□φ), and the converse Barcan formula (∀x□φ → □∀xφ). Interestingly, the system that I have developed for the modal logic K, when the standard rules for quantification are added can prove the converse Barcan formula, but not the Barcan formula. However, moving to a stronger system such as S4 allows the Barcan formulas to be proven.

Other interesting features of the modal systems conforming to my philosophy of logic. For instance, there is no cut free formulation of S5. In order to get a cut free system it is necessary to expand the sequent calculus to a hypersequent calculus. A hypersequent is a set of sequences, of the form ‘...|Γ ⇒ Δ|Γ' ⇒ Δ'|Γ'' ⇒ Δ''|...’. The intuitive reading of hypersequents is reading the ‘|’ as a disjunction. The above would be read as “...either Γ ⇒ Δ is provable or Γ' ⇒ Δ' is provable, or Γ'' ⇒ Δ'' is provable, or ...”. I will explore whether hypersequents are acceptable given the
philosophy of logic hitherto explored. If not, I will have to conclude that S5 is not an acceptable modal logic, since it cannot admit of a cut free formulation, or find a cut free system that is acceptable.

There is, as above, a philosophical issue with understanding what a modal operator is when logic is grounded syntactically as opposed to semantically. Since modal operators are not short-hand for talk about possible worlds, I must give an account of what they are that is consistent with the proof-theoretic semantics that I want to give for them. I will argue that it is a mistake to think of modal operators as first being shorthand for possible worlds. This may be a useful heuristic, but it should not be taken seriously. I will take my cue here from the Carnap of *Logical Syntax of Language* [17], as well as Sellars.

### 6.2 Gentzen Style Modal Logic

This section will discuss the need for a proof theory of modal logic if it is to be logic defined in the sense described in the previous chapter. At this point, there are no Gentzen style proof systems that meet the above requirements [100]. Here I will discuss the limitations of a Gentzen style proof system to express modal notions.

Every modal system that has been developed depends on the $k$-rule, which can be formulated in one of two ways:

$$k_1 \quad \frac{\Gamma \Rightarrow \varphi, \Sigma}{\Box \Gamma \Rightarrow \Box \varphi, \Diamond \Sigma}$$

$$k_2 \quad \frac{\Gamma \Rightarrow \varphi}{\Box \Gamma \Rightarrow \Box \varphi}$$

The restriction on each rule is that the succeedent cannot be empty, and in $k_2$, it must have exactly one element. If the succeedent were empty the resulting system
would have a model-theory that required the relation between worlds to be serial. These rules characterize the familiar System K. Extensions of this system are had by adding further rules that will be discussed at length in this section. This, however, is problematic as none of these rules can be counted as determining the meaning of the logical connectives. $k_1$ introduces both $\Box$ and $\Diamond$, $k_2$ does not define the connective in a way that determines what it is to assert a modal sentence when other modality is not present. In particular, the problem with both of these is that they prevent the reduction of the structural rule Weak [37].

In addition to this, there is no cut-free formulation of the modal logic S5 [59]. All of this is problematic as it rules out modality as being a logical notion. This is a bullet that I do not want to bite, so it is necessary to find a way to expand the sequent calculus to capture modality.

An important point to note though is that when most modal are combined with the natural rules for quantification result in a system that is a hybrid between contingentist and necessitist. As I said above, the necessitist claims roughly that any object that exists necessarily exists, the contingentist denies this. What many have taken to characterize the necessitist view is their adherence to the Barcan formula, $(\Box \forall x \varphi \rightarrow \forall x \Box \varphi)$, and the converse Barcan formula $(\forall x \Box \varphi \rightarrow \Box \forall x \varphi)$. Interestingly, it is only when a symmetry axiom (or something implying it) along standard rules for quantification are added can the Barcan formula be proved. Otherwise, the systems only prove the converse Barcan formula.[43]. Without an axiom corresponding to the symmetry restriction on models the Barcan formula is not provable in the system.

This is an important result since it shows the philosophical importance of the ap-
proach that is taken to logic. As noted above, the Barcan formulas are a crucial part of the debate between necessitism and contingentism. If it turns out that approaching modality in this way cannot justify the Barcan formula, then this will count as a mark against the necessitist view. This is one potential philosophical result from approaching logic and modality in the way that I have described.

6.3 Hypersequents and Metasequents

One way to get something like a sequent calculus formulation of modal logic but retain cut eliminability for S5 is to move to a hypersequent calculus. This system is defined by inference rules that move from set(s) of sequents to a sequent. Let $G$ and $H$ be sequents, then $G; H$ is a hypersequent. The intuitive reading of ‘;’ is as a disjunction. Moving to a hypersequent calculus makes cut eliminable for S5.

This is an improvement on the above, but it is not sufficient, since the system still relies on the non-meaning conferring $k$ rule. A step in a more promising direction was made by Došen [23], whose system is the forerunner of what I will call the metasequent calculus. His system is characterized by the fact that it includes a hierarchy of metalanguages that characterize arguments that are valid in the object languages they are about.

A metasequent is a pair of sets of sequents. $^{13}G \vdash H$, where $G$ and $H$ are sets of sequents. A calculus for a metasequent system is a series of inferences that outline introduction rules for the connectives in a language. So a metasequent is a generalization on the notion of a hypersequent, and thus can provide a cut-free formulation of S5.

$^{13}G \vdash H$, where $G$ and $H$ are sets of sequents.
An even more attractive feature of this system include two rules for the modal operator:

\[
\frac{G; \vdash \varphi \vdash \Gamma \Rightarrow \Sigma; H}{G; \vdash \Gamma, \Box \varphi \Rightarrow \Sigma; H} \quad \frac{G; \vdash \Gamma \Rightarrow \Sigma; H}{G; \vdash \varphi \vdash \Gamma \Rightarrow \Sigma; H}
\]

As Došen proves, these rules will characterize the system S4, when the metasequent is only allowed to have a single conclusion, and S5, if the metasequent is multiple conclusions. While this approach does set out rules that are much closer to meeting the above requirements, it seems only to be able to characterize S4 and S5. Furthermore, it does not allow for the reduction of the rule Weak.

This motivated the search for rules that would be flexible enough to capture many of the important systems of modal logic, but that could meet the above requirements.

7 The Metasequent Calculus

7.1 The Calculus

This section will present the metasequent calculus and the proposed rules governing the introduction of connectives. At the moment, the goal is to have most of the work of accounting for the different modal systems done by manipulating structural rules in the calculus. In particular, the validity of the following rule is one that will play a central role in the investigation:

\[
\text{FL}\,?? \quad \frac{G; \vdash \varphi \vdash \Gamma \Rightarrow \Sigma; H}{G \vdash \Gamma, \varphi \Rightarrow \Sigma; H}
\]
This is because, depending on the restrictions on the ?? rule, it may be possible to have uniform introduction rules for modal operators, but change what counts as a valid metasequent, when combining different modal operators. There are two crucial points to notice about the rule at the moment.

1. Došen’s rule can be seen as a commitment to the reflexivity axiom and unrestricted FL??:\(^{14}\)

\[
\begin{align*}
\text{Ref} & : G; \varphi \vdash \Gamma \Rightarrow \Sigma; H \\
\text{FL??} & : G \vdash \Gamma, \Box \varphi \Rightarrow \Sigma; H
\end{align*}
\]

2. If we allow unrestricted use of the FR?? rule and unrestricted BL\(\Box\), we can derive the Triv axiom:

\[
\begin{align*}
\text{Cut} & : \varphi \Rightarrow \Box \varphi \\
\text{BL\(\Box\)} & : \Box \varphi \Rightarrow \Box \neg \varphi \\
\text{BL\(\Box\)} & : \Box \neg \varphi \Rightarrow \Box \neg \neg \varphi \\
\text{FR??} & : \Box \neg \varphi \Rightarrow \Box \neg \Box \varphi \\
\text{FR??} & : \Box \neg \Box \varphi \Rightarrow \Box \neg \neg \varphi \\
\end{align*}
\]

A first point to note is that for each connective there are four introduction rules for it. This is because each connective can appear in one of four places in a metasequent. It can appear in the antecedent of a sequent in the antecedent of the metasequent (BL), in the succedent of a sequent in the antecedent of a metasequent (BR), in the antecedent of a sequent in the succedent of a metasequent (FL), or in the succedent of a sequent in the succedent of a metasequent (FR). So a metasequent system will have

\(^{14}\)The application of Ref in this proof corresponds to the reflexivity axiom for the back of a metasequent.
to give an introduction rule for each position in the metasequent. This means that in general metasequent systems are more cumbersome, but they allow us to express things that cannot be expressed in the language of sequents alone. For example, the deduction theorems are now theorems of the metasequent system:

1. $G; \Gamma, \varphi \Rightarrow \psi, \Sigma \vdash G; \Gamma \Rightarrow \varphi \rightarrow \psi, \Sigma$

2. $G; \Gamma \Rightarrow \varphi \rightarrow \psi, \Sigma \vdash G; \Gamma, \varphi \Rightarrow \psi, \Sigma$

Here is a portion of the metasequent calculus. There are four conjunction rules.

\[
\begin{align*}
\text{BL} & \quad \frac{G; \Gamma, \varphi, \psi \Rightarrow \Sigma \vdash H}{G; \Gamma, \varphi \land \psi \Rightarrow \Sigma \vdash H} \\
\text{BR} & \quad \frac{G; \Gamma \Rightarrow \varphi_1, \Sigma \vdash H}{G; \Gamma \Rightarrow \varphi_0 \land \varphi_1, \Sigma \vdash H} \\
\text{FL} & \quad \frac{G \vdash \Gamma, \varphi_i \Rightarrow \Sigma; H}{G \vdash \Gamma, \varphi_0 \land \varphi_1 \rightarrow \Sigma; H} \\
\text{FR} & \quad \frac{G \vdash \Gamma \Rightarrow \varphi, \Sigma; H}{G, G' \vdash \Gamma' \Rightarrow \varphi \land \psi; H, H'}
\end{align*}
\]

### 7.2 Results about the Calculus

Here I will prove the results that have not yet been proven about the metasequent calculus. In particular I hope to show that cut elimination holds for this system, and that the connectives meet the requirements outlined in §5. At this point I am unsure if the extensional connectives meet those requirements. I do not see why they would not at the moment, but this also will be an area of research. If they do not, then it will be necessary to carefully reconsider both the requirements on what counts as logical terminology, and the motivation for the metasequent calculus.
If it turns out that there are no modal rules meeting the above requirements, but that those requirements are plausible. It may be necessary to conclude that modality is not a logical notion.

7.3 Philosophical Interpretation of The Metasequent Calculus

If the metasequent calculus does offer a system of modal logic that fits the above requirements I will discuss the philosophical implications of that here. In particular, the success of the metasequent calculus would indicate, or at least be compatible with, modality being something that is ultimately metalinguistic. Meaning for the modal connectives can only be determined in a metalanguage for the sequent calculus, and so the modal connectives have something like a metalinguistic status. In particular, they will mark the relations between assertions and denials of sentences across contexts.

I will discuss Carnap’s system in *Logical Syntax of Language*, and Sellars’s similar approach to modality [88]. Both considered modal claims to be claims *about* the language that we are speaking. Modal claims were ultimately claims that were made in the metalanguage, as is the case with the modal claims in the metasequent calculus. This approach to modality arises naturally in a proof-theoretic context.

This, if successful would also provide an alternative to model-theoretic semantics. It could give philosophers an account of modality that does not require discussion of possible worlds. Furthermore, it provides a reasonable epistemology of modality. Knowing the rules that govern the logical connectives of a language, and moving up to a metalanguage accounts for knowing how modal operators behave.
8 Necessitism and Contingentism

8.1 Second-Order Modal Logic

Here I will combine the second-order logic developed in SS2-4, with the modal logics developed in SS6-7. Depending on the results of the previous section, it may be possible to rule out certain logics based on the impossibility of giving well-behaved introduction rules for them. One logic that will come under close scrutiny in this section is S5. Once these logics have been combined, I will consider some theorems of the system of second-order modal logic. Then I will turn to the metaphysical results of such a system of modal logic.

Recently Williamson has used second-order modal logic to argue that necessitism is true [101, 105]. As mentioned above, necessitism is the view that necessarily everything is necessarily something. Notice that if this is true, there it is not true that there might be something that might be distinct from everything. So there may be possible objects that are not actual. I am expecting my system to give an answer as to whether or not necessitism is true. There will be two places that where I will test where the developed system should provide answers.

8.2 Barcan Formulas

The first place that I will consider answers that my modal logic will give to metaphysical questions is the Barcan formulas. Here I will develop Williamson’s arguments in depth, and carefully explore the second-order modal systems that he develops. From there it will be possible to evaluate his arguments. The first, and most obvious use
is to compare my to the system that Ruth Barcan Marcus developed in [3]. After discussing Marcus’s views, I can look at the recent work of Timothy Williamson [101, 104] and engage with his arguments using modal second-order logic. He argues that while the first-order Barcan formulas may be explained away with a variable domain semantics, this is much more difficult to do in the second-order case. An answer to the first-order case will provide insight into the nature of possible individuals, in particular whether there are any strictly possible individuals. An answer in the second-order case will say something about what possible properties are like. Whether properties at other possible worlds are the same as properties in the actual world, etc.

Ultimately, I will want to give a different interpretation of ‘possible world’ than is often given. However, it will be useful to put the discussion in terms of the debate that has been going on, and then translate it into the notion of possibility that is developed in §7.3.

8.3 ‘Necessarily everything is necessarily something’

The second place that I will look as to whether or not the system of modal logic developed has anything to say about the sentence that Williamson takes to characterize the debate between necessitism and contingentism: ‘Necessarily everything is necessarily something’. [101, 32]15

Once there is an answer given to both questions it will be possible to see how the answers differ or not from Williamson’s, and the reasons for which there is or is not

\[\square \forall x \exists y (x = y)\]
a difference between our answers.

9 Gödel’s Ontological Proof

Finally, the developed system will be applied to Gödel’s ontological proof [36]. This is a second area of metaphysics that has been explored using second-order modal logic, and so it will be worthwhile to explore not only the formal machinery here, but also the philosophical results. Furthermore, because the ontological attempt is an attempt to establish the existence of a the being with all the positive qualities, I can explore whether or not the notion of existence developed in §4.

If this argument does go through, it will be necessary to interpret what exactly the results of the argument are on my view. In particular it will be necessary to explore the plausibility of the premises and explore what sort of being the various definitions used define. If it does not go through, then this is a result that is also worth discussing.

This issue has been explored by Roy Cook [20], and Alan Hazen [38]. I plan to add to this discussion. This will be another application of the development of a system of second-order modal logic that is acceptable given the constraints that I lay down on what a logic can be.
References


